Q4

(a) $y^2(6-x) = x^3, (2, \sqrt{2})$.

Implicitly differentiating both sides with respect to x taking 'y' as a function of x.

$$6y^2 - xy^2 = x^3$$

$$\cancel{6y^2 - xy^2 - x^3 = 0}.$$

$$\frac{\partial}{\partial x}(2x^2 - xy^2) = \frac{\partial}{\partial x}(x^3).$$

$$4x - \left(x\frac{\partial y}{\partial x} + \frac{\partial x}{\partial x}y^2\right) = 3x^2.$$

$$4x - x\frac{\partial y}{\partial x} + y^2 = 3x^2.$$

$$\cancel{-x\frac{\partial y}{\partial x}}\ 4x - 3x^2 + y^2 = x\frac{\partial y}{\partial x}.$$

$$\frac{\partial y}{\partial x} = \frac{4x - 3x^2 + y^2}{x}.$$

$$\left.\frac{\partial y}{\partial x}\right|_{(2,\sqrt{2})} = \frac{4(2) - 3(2)^2 + (\sqrt{2})^2}{2} = \frac{8 - 12 + 2}{2} = -1$$

Slope $= -1$

Equation of tangent is

$$y - y_1 = m(x - x_1)$$

$$y - \sqrt{2} = -1(x - 2)$$

$$y - \sqrt{2} = -x + 2.$$

$$y = -x + 2 + \sqrt{2}$$

(b) $x^2 + 2xy + 4y^2 = 12, \ (2,1)$.

$$\frac{\partial}{\partial x}(x^2 + 2xy + 4y^2) = \frac{\partial}{\partial x}(12).$$

$$2x + 2\left(\frac{\partial y}{\partial x}x + \frac{\partial x}{\partial x}y\right) + 4\frac{\partial y^2}{\partial y} \times \frac{\partial y}{\partial x} = 0.$$

$$2x + 2\left(y + x\frac{\partial}{\partial x}(y)\right) + 8y\frac{\partial}{\partial x}(y) = 0.$$

$$2x + 2y + 2x\frac{\partial}{\partial x}(y) + 8y\frac{\partial}{\partial x}(y) = 0.$$

$$\frac{\partial}{\partial x}(y)\left[2x + 8y\right] = 2x + 2y.$$

$$\frac{\partial y}{\partial x} = \frac{2x+2y}{2x+8y} = \frac{2(x+y)}{2(x+4y)} = \frac{x+y}{x+4y}$$

$$\left.\frac{\partial y}{\partial x}\right|_{(2,1)} = \frac{2+1}{2+4(1)} = \frac{3}{6} = \frac{1}{2}. = \text{Slope}$$

~~Tangent of tk~~ Equation of tangent

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 2)$$

$$y - 1 = \frac{1}{2}x - 1$$

$$y = \frac{1}{2}x - 1 + 1 = \frac{1}{2}x.$$

$$y = \frac{1}{2}x.$$

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